

# Initial state computation for steady-state analysis of induction motor fed by voltage inverter

Jérôme Cros, Maxim Bergeron, Philippe Viarouge and Mehdi Taghizadeh Kakhki

LEEPCI, Laval University, 1065, avenue de la Médecine, Québec, G1V 0A6, CANADA, jerome.cros@gel.ulaval.ca

The starting transients in time-stepping Finite Element Method (FEM) are an issue when we are interested in steady state waveforms to compute the copper and magnetic losses. The time required to reach the steady-state currents can be very long with a voltage supply of the windings. However, if the initial state variables are known, there are no starting transient. This paper presents an efficient method to compute the initial conditions required for a FEM steady-state analysis. The method exploits a high order coupled circuit model where self and mutual inductances are identified by linear magnetostatic FEM. The simulations with this model are very fast and we can evaluate the steady-state currents of an induction machine, in a few seconds. Each massive conductor in the rotor must be split in several circuits to approximate the skin effect. The initial current and rotor position values are then applied to the FEM magnetodynamic simulation and it is possible to obtain the steady-state results with the simulation of only one period. The steady state computation method is validated by the study of a railway bogie made of two 350 HP induction traction motors supplied by the same GTO voltage inverter.

*Index Terms*— AC Machines, Finite element analysis, Motor drives, Power systems transients.

## I. INTRODUCTION

THE MODELLING METHODS using a time stepping simulation for the finite element analysis of cage induction motor fed by static frequency converters are well-known [1], [2]. The main problem is generally the long simulation time required to perform a steady state analysis. The initial transients (currents, torque, etc) are difficult to avoid since we do not know the initial motor state. Depending on the rotor squirrel cage material (copper vs aluminum), electrical time constants can reach several seconds. When the speed is fixed, there are large torque and current oscillations in the transient mode. We have to wait for the steady-state operation in order to compute efficiency and motor losses. The FE simulation time then becomes unacceptable as it may take several hours to obtain a single steady state operation point. In order to reduce the simulation time, some authors proposed to initialize the simulation with a sinusoidal voltage waveform at initial states and then apply a custom variable time-stepping method with the PWM voltage [3]. This method is efficient only when the PWM frequency is high and the current ripple is small.

In this paper, we present a fast method to estimate the steady-state motor operation based on the identification of a coupled circuit. First, we neglect the magnetic saturation and we compute the self and mutual inductances of all coupled circuits in magnetostatic with a 2D FE method for different rotor positions [4]. Then, using Matlab-Simulink, we rapidly solve the coupled circuits including the external connections. All steady state currents are obtained in few seconds. The values of the steady state currents and rotor initial position are then applied in the FE model. Finally, to determine the initial motor state, a magnetostatic computation is performed before starting the simulation with the inverter voltage waveforms.

This method has been developed to analyse the steady state operation of a railway bogie made of two 350 HP induction traction motors supplied by the same GTO voltage inverter. The squirrel rotor cage is made with large section of

rectangular copper bars (Fig.1). Comparisons of the current waveforms have been used to validate the steady state model.

## II. COUPLED CIRCUIT MODEL

A coupled circuit is a simplified approach that can be applied to machines analysis [4]. The coupling between the windings is expressed with inductances when magnetic saturation is neglected. The behavior of an induction motor can be modelled by considering several electrical circuits; one per each stator phase and several per each rotor bar. Indeed, each massive bar must be modelled using several elementary circuits carrying different currents. Circuits of a same bar are connected in parallel. Such method can estimate the circulating current inside the real bar.

One can also add external circuit connexions as rotor ending and stator winding arrangement (star or delta). Each electrical circuit is modeled by a resistance and several inductances that depend on the angular rotor position  $\theta$ .

$$[V] = [R][I] + \frac{d\{[L(\theta)][I]\}}{dt} \quad (1)$$

The current response is computed by (2) assuming that the voltage of each rotor bar is null. This equation is a bit more complex if we consider the end-ring circuit [4].

$$\frac{d[I]}{dt} = [L(\theta)]^{-1} \cdot \left[ [V] - \left( [R] + \Omega \frac{d[L(\theta)]}{d\theta_m} \right) [I] \right] \quad (2)$$



Fig. 1. 350 HP Railway traction motor (rotor & stator).

### III. INDUCTANCE CURVES IDENTIFICATION

The considered motor (Fig.1) has three stator phases, 60 stator slots and 52 rotor copper bars. It has a periodicity on half the machine domain. For this traction motor application, we used 3 circuits for each bar. Each massive rotor bar was divided into three equal sections. It is assumed that the current density in each section is constant. Consequently, the FE method has been used to evaluate the flux linkage between 81 windings (3 stator phases and 78 parts of rotor bars on half rotor domain) with linear magnetostatic simulations for 180 discrete rotor positions (1 position per mechanical degrees). There are 180 tables of 81x81 elements to identify.

To minimize the time of the inductance identification, it is more efficient to use a Gauss elimination method for the matrix inversion. All winding sources are in the second member and the identification problem is solved with only one matrix inversion per each rotor position.

### IV. COMPUTATION WITH THE COUPLED MODEL

The coupled model is solved using classical integration techniques with a variable time-step and linear interpolation inside the inductance curves for any rotor position [4].

The simulation is performed with Matlab-Simulink by supplying the experimental voltage waveforms on the machine terminals. This simulation can be done in 60s for 1s of real working time. Fig.2 shows the current transients in the 3 circuits of a same rotor bar and a comparison of the total bar current with the result of FE simulation taking account of the skin effect. One can verify that the current transients in the 3 sections are different and this justifies the use of the 3 circuits per each bar. Such circuit configuration provides a very good estimation of the total current waveform and is highly efficient to take account of slot harmonics [4].

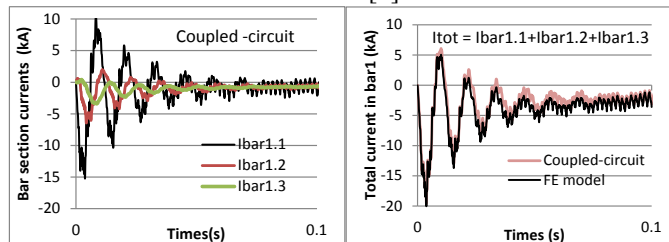


Fig. 2. Rotor bar currents in different parts of a same bar and comparison of the total current in a bar with the FE simulation

### V. STEADY STATE SIMULATIONS WITH 2D FE MODEL

The initial conditions (81 values of current and one rotor position) for the FE simulation are known and we can perform a magnetostatic simulation to compute the initial magnetic solution (magnetic potential in the nodes). This initial solution is used to start the time-stepping FEM without transient.

Fig. 4 presents the first period of the inverter current waveforms (total of 2 motors) obtained at high speed with full-wave voltage (Fig. 3). Fig. 6 is also the first period of the current shapes at low speed with the experimental PWM voltage waveforms of Fig. 5. There are no transients and the steady state shape is quite well preserved. The losses are easily estimated with only one simulation period (stator or rotor).

Consequently, the proposed method is really interesting as it ensures a fast steady state analysis with transient FEM.

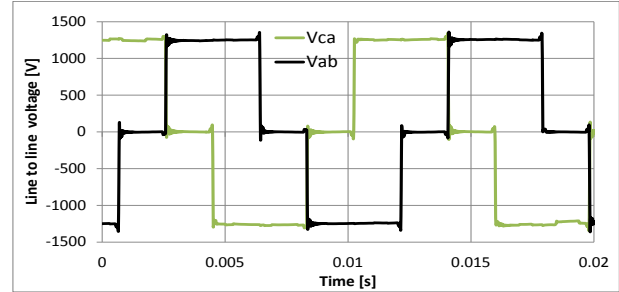


Fig. 3. Experimental line to line stator voltage at high speed (2555 rpm).

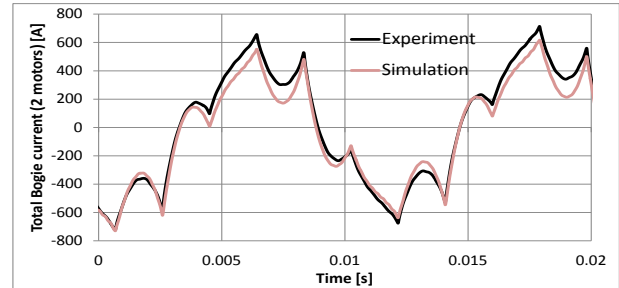


Fig. 4. Comparison of total bogie current waveforms (2555 rpm).

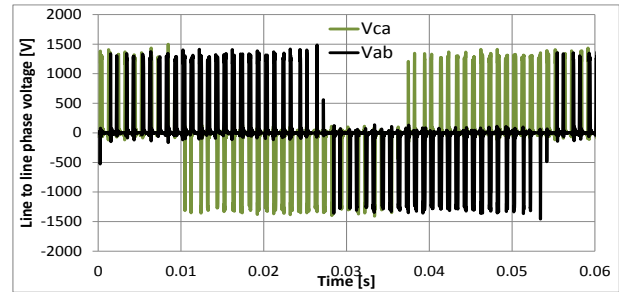


Fig. 5. Experimental line to line stator voltage at low speed (535 rpm).

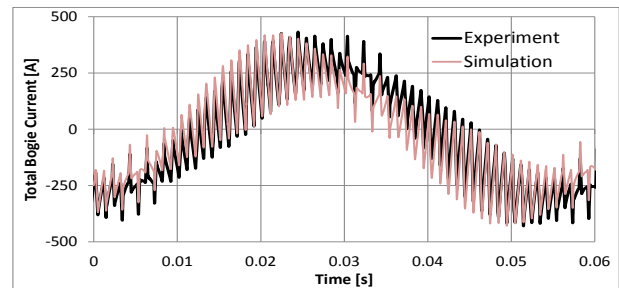


Fig. 6. Comparison of total bogie current waveforms (535 rpm).

### REFERENCES

- [1] A. Arkkio, "Finite Element Analysis of cage induction motors fed by static frequency converters," *IEEE Trans. Magn.*, vol.26, No. 2, pp. 551-554, March 1990.
- [2] T.H. Pham, P.F. Wendling, S.J. Salon, H. Acikgoz, "Transient Finite Element Analysis of an induction motor with external circuit connections and electromechanical coupling," *IEEE Trans. Magn.*, vol.14, No. 4, pp. 1407-1412, December 1999.
- [3] J.J. Lee, Y.K. Kim, H. Nam, K.H. Ha, J.P. Hong, D.H. Hwang, "Loss distribution of three-phase induction motor fed by pulsewidth-modulated inverter," *IEEE Trans. Magn.*, vol.40, No. 2, pp. 762-765, March 2004.
- [4] J. Mathault, M. Bergeron, S. Rakotovololona, J. Cros, P. Viarouge, "Influence of discrete inductance curves on the simulation of a round rotor generator using coupled circuit method," *Electrimacs 2014 conf.*, Valencia, 19-22 May 2014.